Solutions to RSPL/1 (Basic)

1. (*d*), The given equations are

$$4x + py = 21$$
$$px - 2y = 15$$

Since, the given equations have unique solution

$\therefore \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \quad \frac{4}{p} \neq \frac{p}{-2}$

$$p^2 \neq -8 \Rightarrow p \neq \sqrt{-8}$$

So, given system of equation is consistent with unique solution for all values of p other than $\sqrt{-8}$.

Option (*d*) is correct.

2. (*c*), $3x^2 - 2x + 3 = 0$

$$\text{Discriminant (D)} = b^2 - 4ac$$
$$= (-2)^2 - 4 \times 3 \times 3$$
$$= 4 - 36 = -32 < 0$$

$\therefore$ No real roots

Hence, option (*c*) is correct.

3. (*c*), Given: Area of circle = $2 \cdot$ circumference of circle

$\Rightarrow \quad \pi r^2 = 2(2\pi r)$

$\Rightarrow \quad r = 4$

Then diameter is $2r = 2 \times 4 = 8$ units

$\therefore$ Option (*c*) is correct.

4. (*c*), If two figures have same shape but not necessarily the same size, then they are similar figures.

$\therefore$ Option (*c*) is correct.

5. (*a*), Let edge of new cube is x

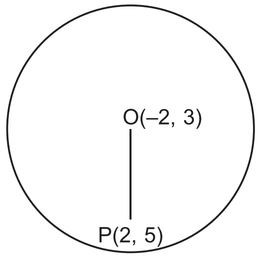
$$\text{Volume of new cube} = \text{volume of 3 cubes melted}$$
$$x^3 = (6) + (8)^3 + 10^3$$
$$x^3 = 216 + 512 + 1000$$
$$x^3 = 1728$$
$$x^3 = 12 \times 12 \times 12$$
$$x^3 = (12)^3$$

$\therefore \quad x = 12$

$\therefore$ Edge of new cube formed is 12 cm

$\therefore$ Option (*a*) is correct.

6. (*b*), Let O(–2, 3) be the center and P(2, 5) is any point



$$OP = \sqrt{(2+2)^2+(5-3)^2}$$
$$= \sqrt{16+4}$$
$$= \sqrt{20}$$
$$OP = 2\sqrt{5}$$
$$OP < 5$$

∴ The point P lies inside the circle.

∴ Option (*b*) is correct.

7. (*d*), Here $a = 5$, $d = 4$, $n = 20$

$$s_{20} = \frac{20}{2}[2 \times 5 + (20 - 1)\cdot 4]$$
$$= 10(10 + 76) = 860$$

∴ Option (*d*) is correct.

8. (*c*), ∵ $\frac{5}{2}$ is a root (zero) of $p(x) = 2x^2 - 8x - m$

$$\Rightarrow \quad 2\left(\frac{5}{2}\right)^2 - 8\left(\frac{5}{2}\right) - m = 0$$
$$\Rightarrow \quad m = \frac{25}{2} - 20$$
$$m = \frac{-15}{2}$$

Now $$p(x) = 2x^2 - 8x + \frac{15}{2}$$
$$= \frac{1}{2}(4x^2 - 16x + 15)$$
$$= \frac{1}{2}(4x^2 - 6x - 10x + 15)$$
$$= \frac{1}{2}[2x(2x - 3) - 5(2x - 3)]$$
$$= \frac{1}{2}(2x - 3)(2x - 5)$$

For roots of polynomial, $p(x) = 0$

$$\Rightarrow \quad \frac{1}{2}(2x - 3)(2x - 5) = 0 \Rightarrow x = \frac{3}{2}, x = \frac{5}{2}$$

⇒ The other zero is $\frac{3}{2}$.

∴ Option (*c*) is correct.

9. (*a*) $\angle OAB = 30°$

PA = PB, $\therefore$ $\angle PAB = \angle PBA$

In ΔAPB

$\Rightarrow$ $\angle APB + \angle PAB + \angle PBA = 180°$

$\Rightarrow$ $60° + 2\angle PAB = 180°$

$\Rightarrow$ $\angle PAB = 60°$

$OA \perp PA$,

[$\because$ tangents and radius are perpendicular to each other]

$\therefore$ $\angle OAP = 90°$

$\angle OAP = \angle OAB + \angle PAB = 90°$

$\Rightarrow$ $\angle OAB = 90° - 60°$

$\therefore$ $\angle OAB = 30°$

10. (*a*), $2\pi R = 2\pi R_1 + 2\pi R_2$

$\Rightarrow$ $2\pi R = 2\pi(R_1 + R_2)$

$\Rightarrow$ $R = (R_1 + R_2)$

$\therefore$ Option (*a*) is correct.

11. p, q, r in A.P.

$\Rightarrow$ $2q = p + r$ ...(*i*)

$\Rightarrow$ To find $p^3 + r^3 - 8q^3 = p^3 + r^3 - (2q)^3 = p^3 + r^3 - (p + r)^3$ [from (*i*)]

$= p^3 + r^3 - p^3 - r^3 - 3pr(p + r)$

$= - 3pr(2q)$

$= - 6pqr$

12. Given: $P(E) + P(\overline{E}) = q$

$\because$ $P(E) + P(\overline{E}) = 1$

13. 0 or 1

$\because$ $2^2 = 3 + 1, 3^2 = 9 + 0, 4^2 = 15 + 1, 5^2 = 24 + 1$

14. $t_{n+1} = 6(n + 1) + 5$

$= 6n + 6 + 5$

$= 6n + 11$

15. Given: $\sin\theta \cdot \cos(90° - \theta) + \cos\theta \sin(90° - \theta)$

$= \sin\theta \cdot \sin\theta + \cos\theta \cdot \cos\theta$

$= \sin^2\theta + \cos^2\theta$

$= 1$

OR

$\sin^2 20° + \sin^2 70° = \sin^2 20° + \sin^2(90° - 20°)$ [$\because \sin(90° - \theta) = \cos\theta$]

$= \sin^2 20° + \cos^2 20°$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$= 1$

16. $\because$ Ratio of areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

$\therefore$ Ratio of areas of similar triangles = $\left(\frac{3}{7}\right)^2 = \frac{9}{49}$.

17. The given data is $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$

$$n = 8 \text{ (even), median} = a$$

$$\text{Median} = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}\right] \text{ observation}$$

$$= \frac{1}{2}[4^{\text{th}} + 5^{\text{th}} \text{ observation}]$$

$$= \frac{1}{2}(x_4 + x_5) = a$$

Now median of x_3, x_4, x_5, x_6

Here $n = 4$ (even)

$$\text{Median} = \frac{1}{2}[\text{2nd} + \text{3rd observation}]$$

$\Rightarrow$ $$\frac{x_4 + x_5}{2} = a$$

OR

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$6 = \frac{2p+52}{11}$$

$$66 = 2p + 52$$

$$2p = 14$$

$$p = 7$$

18.

$$2\sin 2\theta = \sqrt{3}$$

$$\sin 2\theta = \frac{\sqrt{3}}{2}$$

$$\sin 2\theta = \sin 60°$$

$$2\theta = 60°$$

$\Rightarrow$ $$\theta = 30°$$

19. The given equation is,

$$\sqrt{3}x^2 + 27x + 5\sqrt{3} = 0$$

Here $a = \sqrt{3}, b = 27, c = 5\sqrt{3}$

$$\text{Sum of zeroes} = (\alpha + \beta) = \frac{-b}{a} = \frac{-27}{\sqrt{3}} = -9\sqrt{3}$$

Product of zeroes = $\alpha\beta = \frac{c}{a} = \frac{5\sqrt{3}}{\sqrt{3}} = 5$

20. $\tan 15° \cdot \tan 20° \tan 70° \tan 75°$

$= (\tan 15° \tan 75°) \cdot (\tan 20° \cdot \tan 70°)$

$= [\tan 15° \tan(90° - 15°)][\tan 20° \cdot \tan(90° - 20°)]$

$= (\tan 15°\cot 15°) \cdot (\tan 20°\cot 20°)$

$= \left(\tan 15° \times \dfrac{1}{\tan 15°}\right) \cdot \left(\tan 20° \times \dfrac{1}{\tan 20°}\right)$

$= (1) \cdot (1) = 1$

21. $7 \times 11 \times 13 + 13 = 13(7 \times 11 + 1) = 13 \times 78$

$= 13 \times 13 \times 3 \times 2$

= Product of more than one prime

∴ This number is composite number.

Also $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

$= 5[7 \times 6 \times 4 \times 3 \times 2 + 1]$

$= 5 \times 1008$ = product of more than one prime

∴ $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ is composite number.

OR

Here $867 = 225 \times 3 + 192$

$225 = 192 \times 1 + 33$

$192 = 33 \times 5 + 27$

$33 = 27 \times 1 + 6$

$27 = 6 \times 4 + 3$

$6 = 3 \times 2 + 0$

∴ HCF = 3

22. Here, PQ = 10 units

$\Rightarrow \sqrt{(10-2)^2 + (-y+3)^2} = 10$

Squaring both sides, we get

$\Rightarrow 64 + (3 - y)^2 = 100$

$(3 - y)^2 = 36$

$3 - y = \pm 6 \Rightarrow y = 9$ or $y = -3$

23. Total number of coins = 100 + 50 + 20 + 10 = 180

Number of 50p coins = 100

(*i*) $p(50p \text{ coin}) = \dfrac{100}{180} = \dfrac{5}{9}$

(*ii*) $p(₹\ 5 \text{ coin}) = \dfrac{10}{180} = \dfrac{1}{18}$

$\therefore p(\text{not a ₹ 5 coin}) = 1 - \dfrac{1}{18} = \dfrac{17}{18}$

50*p* coin has more probability to fall out.

24. The multiples of 4 between 10 and 250 be 12, 16, 20, 24, ... 248

Here, $a = 12, d = 4, a_n = 248$

$a_n = a + (n - 1)\, d$

$\Rightarrow 248 = 12 + (n - 1)\, 4$

$\Rightarrow 248 - 12 = (n - 1)\, 4$

$\Rightarrow \quad \frac{236}{4} = n - 1$

$\Rightarrow \quad 59 = n - 1$

$\Rightarrow \quad n = 59 + 1 = 60$

OR

2-digit numbers divisible by 3 are 12, 15, 18,.....99

Here, $a = 12$, $d = 3$

Let there are n numbers

$\therefore \quad a_n = 99$

$\Rightarrow \quad a + (n - 1)d = 99$

$12 + (n - 1) \times 3 = 99 \Rightarrow (n - 1) \times 3 = 87$

$\Rightarrow \quad n - 1 = 29 \Rightarrow n = 30$

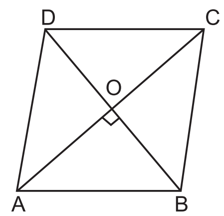
$\therefore$ There are 30 two digit numbers which are divisible by 3.

25. Given: ABCD is a rhombus and AC and BD are its diagonals

To Prove: $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

Proof: Here $\quad AO = \frac{1}{2}AC, \; BO = \frac{1}{2}BD$

(diagonals of rhombus bisect each other at 90°)



In right ΔAOB, $AB^2 = AO^2 + OB^2$

$\Rightarrow \quad AB^2 = \frac{1}{4}AC^2 + \frac{1}{4}BD^2$

$\Rightarrow \quad 4AB^2 = AC^2 + BD^2$

$\Rightarrow \quad AB^2 + AB^2 + AB^2 + AB^2 = AC^2 + BD^2$

$\Rightarrow \quad AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2 \qquad (\because AB = BC = CD = AD)$

26. Here, $\quad \angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle A + \angle B = 180° - C$

$\Rightarrow \quad \frac{\angle A + \angle B}{2} = 90° - \frac{\angle C}{2}$

$\cos\left(\frac{A+B}{2}\right) = \cos\left(90° - \frac{C}{2}\right)$

$\Rightarrow \quad \cos\left(\frac{A+B}{2}\right) = \sin\left(\frac{C}{2}\right)$

27. Let $\sqrt{5}$ is a rational number

$\sqrt{5} = \frac{a}{b}$, where a and b are coprime, $b \neq 0$

Squaring both sides

$5 = \frac{a^2}{b^2} \Rightarrow 5b^2 = a^2 \qquad ...(i)$

$\Rightarrow$ 5 is a factor of $a^2 \Rightarrow$ 5 is also a factor of a

let $a = 5c$, c is some integer.

Substituting in (i), we get

$$5b^2 = 25c^2 \Rightarrow b^2 = 5c^2$$

$\Rightarrow$ 5 is a factor of $b^2 \Rightarrow$ 5 is a factor of b.

But a and b are coprimes

5 cannot be a common factor of a and b.

Hence our supposition is wrong.

$\therefore$ $\sqrt{5}$ is an irrational number.

28.

$$
\begin{array}{l}
x^2 + 3x + 1 \overline{\big)\, 3x^4 + 5x^3 - 7x^2 + 2x + 2 \,\big(}\, 3x^2 - 4x + 2 \\
\qquad\qquad\quad \underline{\underset{-}{3x^4} + \underset{-}{9x^3} + \underset{-}{3x^2}} \\
\qquad\qquad\qquad -4x^3 - 10x^2 + 2x + 2 \\
\qquad\qquad\qquad \underline{\underset{+}{-4x^3} - \underset{+}{12x^2} - \underset{+}{4x}} \\
\qquad\qquad\qquad\qquad 2x^2 + 6x + 2 \\
\qquad\qquad\qquad\qquad \underline{\underset{-}{2x^2} + \underset{-}{6x} + \underset{-}{2}} \\
\qquad\qquad\qquad\qquad\qquad \underline{0}
\end{array}
$$

$\because$ remainder = 0

$\therefore$ $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

29.

Let P(x_1, y_1) and Q(x_2, y_2) are the points of trisection of AB

$\therefore$ AP = PQ = BQ

$\Rightarrow$ AB : BP = 1 : 2

$\Rightarrow$ P divides AB in the ratio 1 : 2

$$\Rightarrow \quad x_1 = \frac{1 \times -7 + 2 \times 2}{1 + 2} = -1$$

$$y_1 = \frac{1 \times 4 + 2 \times (-2)}{1 + 3} = 0$$

$\therefore$ Coordinates of P are (–1, 0)

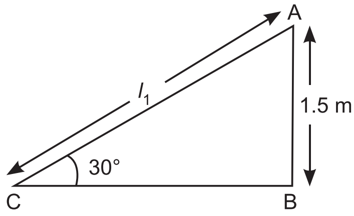
Now Q is mid-point of PB

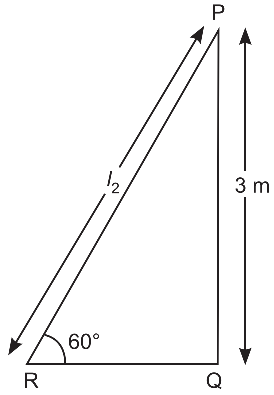
$$\therefore \quad x_2 = \frac{-1 + (-7)}{2} = -4$$

$$y_2 = \frac{0 + (-4)}{2} = 2$$

$\therefore$ Coordinates of Q are (–4, 2)

30. Let l_1 is the length of slide for children below the age of 5 years and l_2 is the length of the slide for elder children.





In ΔABC, $\frac{AB}{AC} = \sin 30°$

$\Rightarrow \frac{1.5}{AC} = \frac{1}{2}$

$\Rightarrow AC = 3m \Rightarrow l_1 = 3m$

In ΔPQR, $\frac{PQ}{PR} = \sin 60°$

$\Rightarrow \frac{3}{PR} = \frac{\sqrt{3}}{2}$

$\Rightarrow PR = \frac{3 \times 2}{\sqrt{3}}$

$= \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3}$

$= 2\sqrt{3}$ m

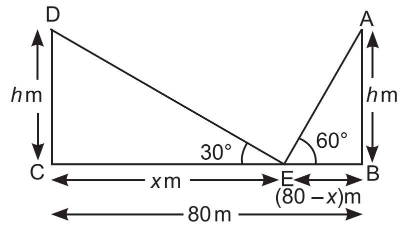
$l_2 = 2\sqrt{3}$ m

OR

Let $AB = CD = h$ m [Height of the poles]

Given: BC = 80 m [Width of the road]

Let CE = x m



$\therefore$ BE = $(80 - x)$ m

In ΔCDE, $\frac{CD}{CE} = \frac{h}{x} = \tan 30°$

$\frac{h}{x} = \frac{1}{\sqrt{3}}$

$\Rightarrow x = \sqrt{3}h$ ... (i)

In ΔABE, $\frac{AB}{BE} = \tan 60° \Rightarrow \frac{h}{80 - x} = \sqrt{3}$

$\Rightarrow h = 80\sqrt{3} - \sqrt{3}x$

$\Rightarrow \sqrt{3}x = 80\sqrt{3} - h$

$\Rightarrow x = \frac{80\sqrt{3} - h}{\sqrt{3}}$ ... (ii)

From equation (i) and (ii), we get

$\sqrt{3}h = \frac{80\sqrt{3} - h}{\sqrt{3}}$

$\Rightarrow 3h = 80\sqrt{3} - h$

$\Rightarrow 4h = 80\sqrt{3} \Rightarrow h = 20\sqrt{3}$

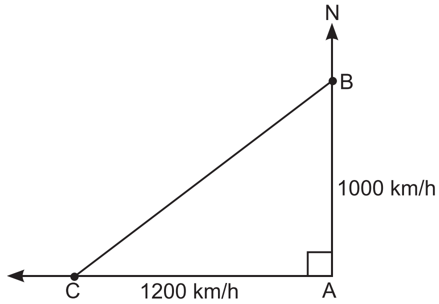
Substituting h in equation (i),

$$x = h\sqrt{3} = 20\sqrt{3}\times\sqrt{3} = 60 \text{ m}$$

Hence, point E is 60 m from pole CD and 20 m from pole AB.

31. Let aeroplanes leave from A.

Plane in the north direction reaches B in $1\frac{1}{2}$ hrs



$\therefore$ $$AB = 1000 \times \frac{3}{2} = 1500 \text{ km}$$

Plane in the west direction reaches at C in $1\frac{1}{2}$ hrs

$\therefore$ $$AC = 1200 \times \frac{3}{2} = 1800 \text{ km}$$

In right ΔBAC, $BC^2 = AB^2 + AC^2$

$\Rightarrow$ $$BC^2 = (1500)^2 + (1800)^2$$

$\Rightarrow$ $$BC^2 = 2250000 + 3240000$$

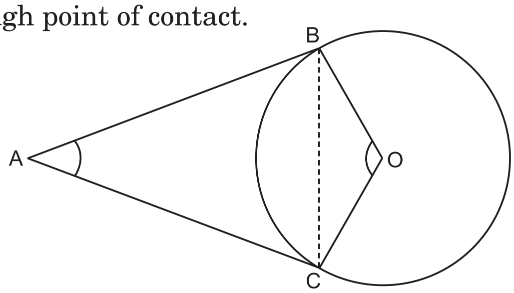
$\Rightarrow$ $$BC^2 = 5490000$$

$$BC = 100\sqrt{549} = 300\sqrt{61} \text{ km}$$

32. Given: AB and AC are tangents to the circle C(O, r) from external point A

To prove: $\angle BAC + \angle BOC = 180°$

$\because$ AB is a tangent and OB is the radius through point of contact.



$\therefore$ $$\angle ABO = 90°$$

Similarly $$\angle ACO = 90°$$

In quadrilateral ABOC

$$\angle BAC + \angle BOC + \angle ABO + \angle ACO = 360°$$

$\Rightarrow$ $$\angle BAC + \angle BOC + 90° + 90° = 360°$$

$\Rightarrow$ $$\angle BAC + \angle BOC = 180°$$

OR

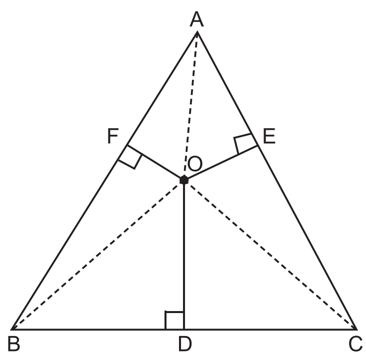
Given: A triangle ABC in which OD $\perp$ BC, OE $\perp$ AC and OF $\perp$ AB

To prove: (i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Construction: Join OA, OB and OC

Proof: (i) in ΔAOF



$$OA^2 = OF^2 + AF^2 \quad \text{[Pythagoras Theorem]}$$

$$AF^2 = OA^2 - OF^2$$

In ΔBDO, $$OB^2 = BD^2 + OD^2$$

$$BD^2 = OB^2 - OD^2$$

In ΔCEO, $$OC^2 = CE^2 + OE^2$$

$$CE^2 = OC^2 - OE^2$$

$\therefore$ $$AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 \quad ...(i)$$

(*ii*) $OA^2 - OF^2 + OB^2 - OD^2 + OC^2 - OE^2 = AF^2 + BD^2 + CE^2$

$(OA^2 - OE^2) + (OC^2 - OD^2) + (OB^2 - OF^2) = AF^2 + BD^2 + CE^2$

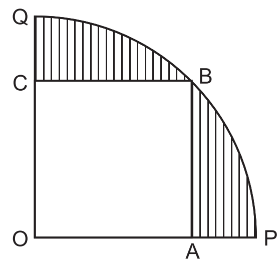
$AE^2 + CD^2 + BF^2 = AF^2 + BD^2 + CE^2$

33. Given: Side of the square OABC = OA = 20 cm

$\therefore$ Area of the square = 20 × 20 = 400 cm^2

Diagonal of the square = $\sqrt{2}$ × (side of the square)

$= \sqrt{2} \times 20 = 20\sqrt{2}$ cm



Radius of the quadrant of circle

= Diagonal of square = $20\sqrt{2}$

Area of quadrant OPBQ = $\dfrac{\pi r^2 \theta}{360°}$

$= \dfrac{3.14 \times 20\sqrt{2} \times 20\sqrt{2} \times 90°}{360°}$

= 314 × 2 = 628 cm^2

Area of the shaded region = Area of the quadrant – Area of the square

= 628 – 400 = 228 cm^2

Unshaded region has more area.

OR

Radius of bucket = 18 cm

Height of bucket = 32 cm

Volume of bucket = $\pi r^2 h = \pi \times 18 \times 18 \times 32 = 10368\,\pi$ cm^3

Let radius of conical heap = r cm

height = 24 cm

Volume of conical heap = Volume of sand in the bucket

$\Rightarrow \quad \frac{1}{3}\pi r^2 \times 24 = 10368\pi$

$\Rightarrow \quad r^2 = \frac{10368}{8}$

$\Rightarrow \quad r^2 = 1296$

$\Rightarrow \quad r = 36$ cm

Let slant height = l cm

$\therefore \quad l^2 = h^2 + r^2$

$l^2 = (24)^2 + (36)^2$

$l = 43.27$ cm

34.

Number of wickets	Class marks (x_i)	Number of bowlers (f_i)	$d_i = x_i - 120$	$u_i = \frac{d_i}{40}$	f_iu_i
20 – 60	40	7	– 80	– 2	– 14
60 – 100	80	5	– 40	– 1	– 5
100 – 140	120 = a (let)	16	0	0	0
140 – 180	160	12	40	1	12
180 – 220	200	2	80	2	4
220 – 260	240	3	120	3	9
Total		$\Sigma f_i = 45$			$\Sigma f_iu_i = 6$

We have,

$$\text{Mean} = a + \frac{\Sigma f_iu_i}{\Sigma f_i} \times h$$

$$= 120 + \frac{6}{45} \times 40$$

$$= 120 + \frac{16}{3}$$

$$= \frac{360+16}{3} = \frac{376}{3} = 125.33$$

On an average a bowler took 125 wickets.

35. Let speed of the stream = x km/hr

Speed of boat in still water = 18 km/h

$\therefore$ Speed downstream = $(18 + x)$ km/h

Distance downstream = 24 km

$\therefore$ Time taken (downstream) = $\frac{24}{18+x}$ hr

Speed upstream = $(18 - x)$ km/h

Distance upstream = 24 km

Time taken upstream = $\frac{24}{18-x}$

A.T.Q.

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$24(18 + x) - 24(18 - x) = 324 - x^2$$

$$x^2 + 48x - 324 = 0$$

$$(x + 54)(x - 6) = 0$$

$\Rightarrow$ $x = -54$ or $x = 6$

Rejecting $x = -54$

Speed of stream = 6 km/hr

OR

$$\frac{1}{x} - \frac{1}{x-2} = 3$$

$$\frac{x-2-x}{x(x-2)} = 3$$

$$\Rightarrow \quad -2 = 3x^2 - 6x$$

$$\Rightarrow \quad 3x^2 - 6x + 2 = 0$$

$$a = 3, b = -6, c = 2$$

$$D = b^2 - 4ac$$

$$= 36 - 4 \times 3 \times 2 = 12$$

$$\therefore \quad x = \frac{6 \pm \sqrt{12}}{2 \times 3} = \frac{3 \pm \sqrt{3}}{3}$$

36. Let the trees be planted 1, 2, 3, 4, 5, ..., 12

Here, $a = 1, d = 1, n = 12$

Total number of trees planted by each section

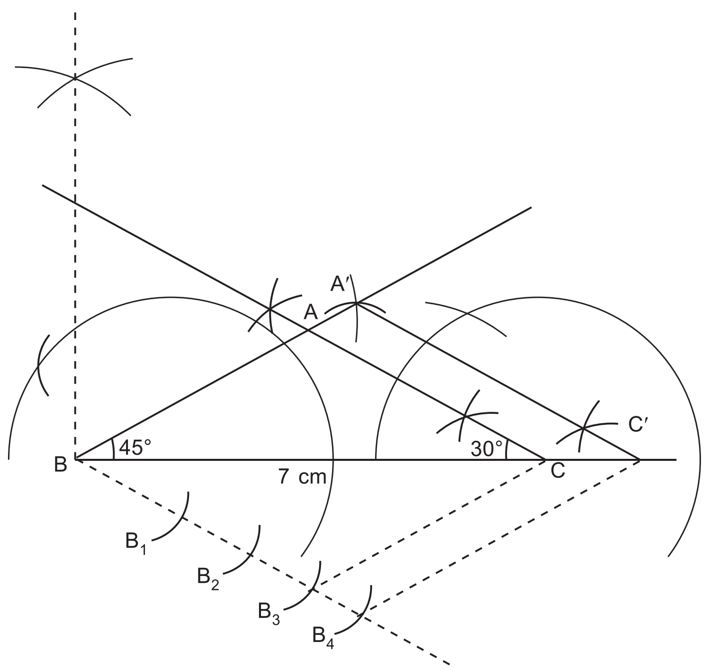
$$S_{12} = \frac{12}{2}[2a + (n-1)d]$$

$$= 6[2 \times 1 + (12 - 1) \times 1]$$

$$= 6[2 + 11] = 6 \times 13 = 78$$

Total number of trees planted by 3 sections = 78 × 3 = 234

37. In ΔABC



$$\angle A + \angle B + \angle C = 180°$$

$$105° + 45° + \angle C = 180°$$

$$\angle C = 180° - 150° = 30°$$

38. A + B = 90° $\Rightarrow$ B = 90° – A

Taking LHS

$$= \sqrt{\frac{\tan A \tan (90 - A) + \tan A \cot (90 - A)}{\sin A \sec (90 - A)} - \frac{\sin^2 (90 - A)}{\cos^2 A}}$$

$$= \sqrt{\frac{\tan A \cot A + \tan A \tan A}{\sin A \operatorname{cosec} A} - \frac{\cos^2 A}{\cos^2 A}}$$

$$= \sqrt{\frac{1 + \tan^2 A}{1} - 1} = \sqrt{\tan^2 A} = \tan A.$$

OR

$$\text{LHS} \quad \frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = \frac{\tan^2\theta}{\tan\theta - 1} + \frac{1}{\tan\theta(1 - \tan\theta)}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{\tan\theta(\tan\theta - 1)}$$

$$= \frac{\tan^3\theta - 1}{\tan\theta(\tan\theta - 1)}$$

$$= \frac{(\tan\theta - 1)(\tan^2\theta + 1 + \tan\theta)}{\tan\theta(\tan\theta - 1)}$$

$$= \frac{\sec^2\theta + \tan\theta}{\tan\theta} = \frac{\sec^2\theta}{\tan\theta} + 1$$

$$= \sec\theta \cdot \operatorname{cosec}\theta + 1 = \text{RHS}$$

39.

Class Interval	Frequency	Cumulative frequency
0 – 100	2	2
100 – 200	5	7
200 – 300	x	$7 + x$
300 – 400	12	$19 + x$
400 – 500	17	$36 + x$
500 – 600	20	$56 + x$
600 – 700	y	$56 + x + y$
700 – 800	9	$65 + x + y$
800 – 900	7	$72 + x + y$
900 – 1000	4	$76 + x + y$
	$76 + x + y$	

$n = 100$

$$\frac{n}{2} = \frac{100}{2} = 50$$

Median = 525

$\therefore$ Median class is 500 – 600

$l = 500, c = 36 + x, f = 20, h = 100$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - c}{f}\right) \times h$$

$$525 = 500 + \frac{50 - (36 + x)}{20} \times 100$$

$$525 - 500 = (50 - 36 - x)5$$

$$\frac{25}{5} = 14 - x$$

$$5 - 14 = -x$$

$$-9 = -x$$

$$x = 9$$

$$76 + 9 + y = 100$$

$$85 + y = 100$$

$$y = 100 - 85 = 15$$

OR

Daily wages	Class marks (x_i)	Number of workers (f_i)	$d_i = x_i - 150$	$u_i = \frac{d_i}{20}$	$f_i u_i$
100 – 120	110	12	– 40	– 2	– 24
120 – 140	130	14	– 20	– 1	– 14
140 – 160	150 = a (let)	8	0	0	0
160 – 180	170	6	20	1	6
180 – 200	190	10	40	2	20
Total		$\Sigma f_i = 50$			$\Sigma f_i u_i = -12$

Here $h = 20$

We have,
$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 150 + \frac{(-12) \times 20}{50}$$

$$= \frac{150}{1} - \frac{24}{5} = \frac{750 - 24}{5} = \frac{726}{5} = ₹\ 145.20$$

40. Radius of lower end (r_1) = 8 cm

Radius of the upper end (r_2) = 20 cm

Height of frustum (h) = 16 cm

Volume of container (frustum) = $\frac{\pi h}{3}[r_1^2 + r_2^2 + r_1 r_2]$

$= 3.14 \times \frac{16}{3}(8^2 + 20^2 + 8 \times 20)$ cm^3

$= 3.14 \times \frac{16}{3}(64 + 400 + 160)$ cm^3

$= 3.14 \times \frac{16}{3} \times 624$ cm^3

$= 10449.92$ cm^3

$= 10.449$ litres

$\left[\begin{array}{ll} \because & 1000\, l = 1 \text{ m}^3 \\ \Rightarrow & 1000\, l = 1000000 \text{ cm}^3 \\ \Rightarrow & \frac{1}{1000} = 1 \text{ cm}^3 \end{array}\right]$

Cost of 1 litre of the milk = ₹ 20

$\therefore$ Total cost = ₹ 20 × 10.449

= ₹ 208.98

Slant height (l) $= \sqrt{(r_2 - r_1)^2 + h^2}$

$= \sqrt{(20-8)^2 + 16^2} = \sqrt{12^2 + 16^2}$

$= \sqrt{144 + 256} = \sqrt{400} = 20$

$\Rightarrow$ $l = 20$ cm

Curved surface area of bucket $= \pi l(r_1 + r_2)$

$= 3.14 \times 20(20 + 8)$ cm^2

$= 3.14 \times 20 \times 28$ cm^2 $= 1758.4$ cm^2

Base area of bucket $= \pi r_1^2$

$= 3.14 \times 8 \times 8$ cm^2

$= 200.96$ cm^2

Metal sheet used = curved surface area of frustum + base area of bucket

$= 1758.4$ cm^2 $+ 200.96$ cm^2

$= 1959.36$ cm^2

Total cost of metal sheet used $= \frac{8 \times 1959.36}{100}$ = ₹ 156.75